40. L. Kaufman and M. Cohen, "Thermodynamics and kinetics of martensite transitions," Usp. Fiz. Metallov, No. 4, 192 (1961).
41. A. Kelly and G. W. Groves, Crystallography and Crystal Defects, Addison-Wesley (1970).
42. A. L. Roitbrud, "Current state of the theory of martensite transitions," in: Investigation of Crystalline Structure and Martensite Transitions [in Russian], Nauka, Moscow (1972).
43. V. N. German, M. P. Speranskaya, L. V. Al'tshuler, and L. A. Tarasova, "Investigation of the structure of monocrystals of iron silicate deformed by strong shock waves," Fiz. Met. Metalloved., 30, No. 3, 1018 (1970).
44. H. Knapp and U. Dehlinger, "Mechanics and kinetics of diffusionless martensite transition," Acta Metallurg., 4, 289 (1956).
45. O. N. Breusov, "Phase transitions caused by shock compression," in: Proceedings of the All-Union Symposium on Pulsed Pressures [in Russian], Vol. 2, Moscow (1974), p. 18.
46. D. E. Grady, W. Y. Murry, and P. S. DeCarly, "Hugoniot sound velocities and phase transformations in two silicates," J. Geophys. Res., 80, No. 5, 4857 (1975).

SHOCK WAVES IN DILATANT AND NONDILATANT MEDIA

S. G. Artyshev and S. Z. Dunin

UDC 622.235.5+539.3+539.214

With the explosion of a charge in an isotropic brittle medium at rest and compressed by a lithostatic pressure p_h, a shock wave starts to propagate outward from the center of the explosion. A step-by-step analysis of the character of the breakdown as a function of the properties of the rock and the lithostatic pressure is given in [1-3], where the breakdown of the rock is described without taking account of the dilatant character of the behavior of the medium, i.e., without taking account of the possibility of a change in the volumetric deformation with shear [4].

The present article discusses the possibility of the propagation of a spherically symmetrical breakdown wave in dilatant and nondilatant plastic media.

The source of the breakdown wave, located in a spherical cavity (cavern) with an initial radius a_0, is a gas having an initial pressure p_{k_0}. It is assumed that the Prandtl plasticity condition is satisfied behind the front of the wave:

$$\sigma_r - \sigma_\varphi = k + m(\sigma_r + 2\sigma_\varphi), \quad (1)$$

where k and m are known constants; σ_r and σ_φ are the stresses in a radial direction and in directions orthogonal to it, respectively. The flow of the rock behind the front is described by the equations of the convservation of momentum and mass and the equation of dilatancy:

$$\rho(\partial u/\partial t + u\partial u/\partial r) = \partial\sigma_r/\partial r + 2(\sigma_r - \sigma_\varphi)/r; \quad (2)$$

$$\partial\rho/\partial t + u\partial\rho/\partial r + \rho(\partial u/\partial r + 2u/r) = 0; \quad (3)$$

$$\partial u/\partial r + 2u/r = \Lambda(\rho, \delta_r)|\partial u/\partial r - u/r|, \quad (4)$$

where ρ is the density of the medium; u is the mass velocity; r is the radius; t is the time; and $\Delta(\rho, \sigma_r)$ is the rate of dilatancy [4]. At the front of the breakdown wave, the conditions of the conservation of mass and momentum are adopted:

$$u_f(t) = \varepsilon_f(t)\dot{R}(t); \quad (5)$$

$$p_f(t) - p_h = \rho_0\varepsilon_f(t)\dot{R}^2(t), \quad (6)$$

where R(t) and $\dot{R}(t)$ are the radius and the velocity of the front; $p_h = 9.81 \cdot \rho_0 h$ is the lithostatic pressure at the depth h; $\varepsilon_f = 1 - \rho_0/\rho_f$ is the discontinuity of the density at the front; and $p_f = -\sigma_f$ is the pressure at the front. Here and in what follows the subscript f denotes values of the quantities at the front, while the subscript 0 denotes values in the unperturbed medium.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 104-108, July-August, 1978. Original article submitted July 12, 1977.

It is convenient to write Eqs. (2)-(4) in Lagrangian variables (r_0, t). Denoting the new unknown functions of the variables (r_0, t) by the same letters u, ρ, σ_r, and r, taking account of (1) we obtain that the following equations are satisfied in the region behind the front [$t > t_1(r_0)$, where $t_1(r_0)$ is a function inverse to the function R(t)]:

$$\frac{\partial}{\partial r_0}\left[r^{\alpha}\left(p-\frac{k}{3m}\right)\right]=\rho_0 r_0^2 r^{\alpha-2}\frac{\partial u}{\partial t}; \tag{7}$$

$$\frac{\partial r}{\partial r_0}=\frac{r_0^2}{r^2}\frac{\rho_0}{\rho}; \tag{8}$$

$$\Lambda\frac{\partial}{\partial t}\ln(\rho r^3)+\frac{\partial}{\partial t}\ln\rho=0, \tag{9}$$

where $\alpha = 6m/(2m + 1)$; $p(r_0, t) = -\sigma_r(r_0, t)$.

The equation of dilatancy (9) was obtained under the assumption that $\partial u/\partial r \leqslant 0$ in (4).

We make two additional assumptions, which allow us to go over from equations in partial derivatives to an ordinary differential equation.

1. The rate of dilatancy Λ is a constant. Then Eq. (9) assumes the form

$$\frac{\partial}{\partial t}(\rho^{\Lambda+1}r^{3\Lambda})=0,$$

which can be integrated explicitly:

$$\rho(r_0,t)=\rho_f(t_1(r_0))\left(\frac{r_0}{r(r_0,t)}\right)^{2-n}, \tag{10}$$

where $n = (2 - \Lambda)/(1 + \Lambda)$.

Integration of (8) gives

$$R^{n+1}(t)-r^{n+1}(r_0,t)=(n+1)\rho_0\int_{r_0}^{R(t)}\frac{\xi^n}{\rho_f(t_1(\xi))}d\xi, \tag{11}$$

from which it follows that

$$u(r_0,t)\equiv\frac{\partial r(r_0,t)}{\partial t}=\varepsilon_f(t)\left(\frac{R(t)}{r(r_0,t)}\right)^n\dot R(t). \tag{12}$$

The rock being broken down attains its limiting compression at the front, i.e., $\varepsilon_f(t)$ = const [5]. In this case, Eq. (11) goes over into

$$r^{n+1}(r_0,t)=(1-\varepsilon_f)r_0^{n+1}+\varepsilon_f R^{n+1}(t), \tag{13}$$

and the formula (10) for the decrease in density of the medium can be written in terms of Euler coordinates:

$$\rho(r,t)=\frac{\rho_f^{\Lambda+1}}{\rho_0^{\Lambda}}\left[1-\varepsilon_f\left(\frac{R(t)}{r}\right)^{n+1}\right]^{\Lambda},$$

where

$$\left((1-\varepsilon_f)a_0^{n+1}+\varepsilon_f R^{n+1}(t)\right)^{\frac{1}{n+1}}\leqslant r\leqslant R(t).$$

We introduce the dimensionless quantities $\tau = t/\beta$, $x_0 = r_0/a_0$, $y(x_0, \tau) = r(a_0x_0, \beta\tau)/a_0$, $Y(\tau) = R(\beta\tau)/a_0$, and $\pi(x_0,\beta\tau) = p(a_0x_0, \beta\tau)/p_{k0}$, where as the unit of time we take

$$\beta=\frac{10}{\sqrt{9.80665}}a_0\sqrt{\frac{\rho_0}{p_{k0}}}$$

(a_0, m; ρ_0, g/cm³; p_{k0}, kbar; β, msec). After integration in the limits from r_0 to R(t), taking account of (12), (13), (5), and (6), Eq. (7) assumes the form

$$A\left(\frac{x_0}{Y}\right)Y\ddot Y+B\left(\frac{x_0}{Y}\right)\dot Y^2+C(x_0,\tau)=0, \tag{14}$$

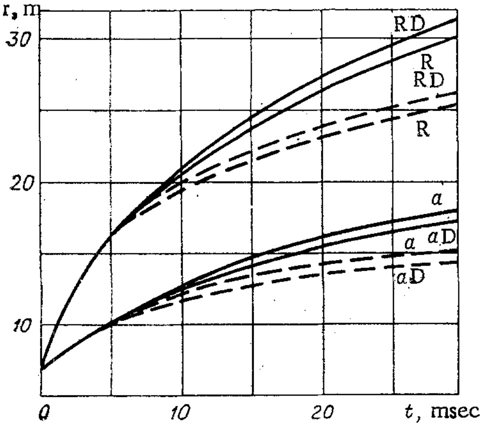

Fig. 1

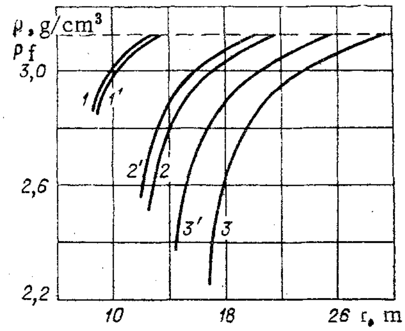

Fig. 2

where

$$A\left(\frac{x_0}{Y}\right)=\int\limits_{x_0/Y}^{1}\xi^2 z^{\alpha-n-2}(\xi)\,d\xi;$$

$$z(\xi)=(\varepsilon_f+(1-\varepsilon_f)\,\xi^{n+1})^{\frac{1}{n+1}};$$

$$B\left(\frac{x_0}{Y}\right)=1+nA\left(\frac{x_0}{Y}\right)-\varepsilon_f n\int\limits_{x_0/Y}^{1}\xi^2 z^{\alpha-2n-3}(\xi)\,d\xi;$$

$$C(x_0,\tau)=\frac{1}{\varepsilon_f}\frac{\rho_0 h}{p_{k0}}10^{-4}-c_1-z^{\alpha}\left(\frac{x_0}{Y}\right)\left[\frac{1}{\varepsilon_f}\pi(x_0,\tau)-c_1\right];$$

$$c_1=\frac{1}{\varepsilon_f}\frac{k}{3mp_{k0}}10^{-3};$$

(k, kg/cm²; h, m).

Setting $x_0 = 1$ and assigning the law of the change in the pressure $\pi(1, \tau)$ in the explosion cavity, Eq. (14) can be used as the differential equation with respect to the dimensionless radius of the front $Y(\tau)$. Under these circumstances, the initial data will be

$$Y(0)=1,\quad \dot{Y}(0)=\sqrt{\frac{1}{\varepsilon_f}\left(1-\frac{\rho_0 h}{p_{k0}}10^{-4}\right)}.$$

After finding $Y(\tau)$, Eq. (14) makes it possible to calculate the pressure $\pi(x_0, \tau)$ over the whole zone through which the wave has passed, $1\leqslant x_0\leqslant Y(\tau)$.

In the present work, the condition in the explosion cavity was taken in two forms:

$$\pi(1,\tau)=(a_0/r(a_0,t))^{3\gamma}\equiv y^{-3\gamma}(1,\tau), \tag{15}$$

which corresponds to the postulation of adiabatic expansion of the cavity with a constant adiabat γ [6], and on the basis of an experimentally established dependence from [7], which, in our notation, has the form

$$\pi(1,\tau)=\left(1+\frac{\beta}{t_r}\tau\right)\exp\left(\frac{1}{2}\left(1-\left(1+\frac{\beta}{t_r}\tau\right)^2\right)\right), \tag{16}$$

where t_r is a constant (msec).

Figures 1-4 give some results of calculations, obtained for the following initial data [7]: p_{k0} = 62 kbar; m = 0.45; k = 0.35 kg/cm²; ρ_0 = 2.5 g/cm³; h = 1000 m; ε_f = 0.2; a_0 = 7 m; γ = 1.5; t_r = 2 msec; and Λ = 0.14 or Λ = 0. The dependencies of the radii of the front R and the cavity a on the time t are given in Fig. 1, where the solid lines correspond to the boundary condition (15) and the dashed lines to the boundary condition (16). Here and in what follows, the letter D relates to the case where the dilatancy is taken into consideration ($\Lambda \neq 0$). The decrease in the density of the broken rock due to dilatancy is illustrated in Fig. 2; the curves 1 and 1' relate to the moment of time 2.7 msec; 2 and 2' relate to 10.8

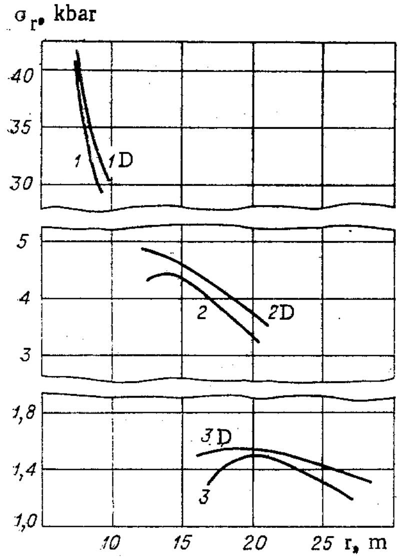

Fig. 3

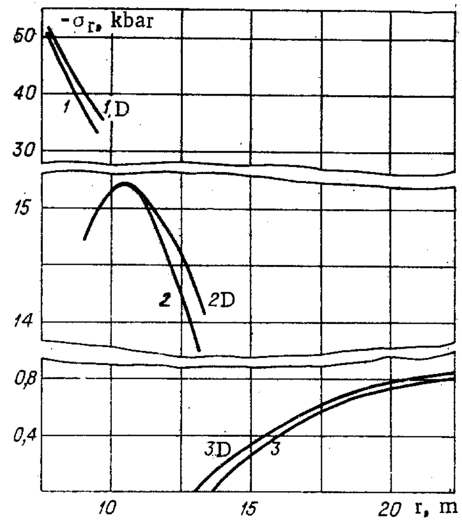

Fig. 4

msec; 3 and 3' relate to 26 msec. Curves 1-3 correspond to the boundary condition (15) and 1'-3' to the boundary condition (16). The radial stress σ_r as a function of the Euler coordinate r is shown in Fig. 3 for boundary condition (15) and in Fig. 4 for boundary condition (16). In Fig. 3, curves 1 and 1D relate to the moment of time 0.9 msec; 2 and 2D relate to 9.9 msec; and 3 and 3D relate to 22.5 msec. In Fig. 4, the corresponding times are equal to 0.9, 2.7, and 16.2 msec.

Calculations showed that, starting from some moment of time, an internal zone of supercompression is formed in the dispersed rock (the maximum on curves 2, 3, and 3D of Fig. 3 and on curves 2, 2D, 3, and 3D of Fig. 4). This phenomenon is observed numerically both taking account and not taking account of dilatancy. As can be seen, taking account of dilatancy leads to a rise in the velocity of the front and to a decrease in the rate of expansion of the cavity (see Fig. 1), as well as to an increase of the pressure in the rock (see Figs. 3 and 4). The decrease in the density due to dilatancy can reach considerable proportions (up to 30% of the pressure at the front, Fig. 2).

Calculations made for other values of p_{k0}, ε_f, γ, and Λ gave approximately the same qualitative result.

The authors wish to express their thanks to B. L. Rozhdestvenskii, E. E. Lovetskii, and V. K. Sirotkin for their evaluation of and interest in the work.

LITERATURE CITED

1. S. S. Grigoryan, "Questions in the mathematical theory of the deformation and breakdown of hard rocks," Prikl. Mat. Mekh., 31, No. 4 (1967).
2. A. B. Bagdasaryan, "Exact solutions of the problem of the action of the explosion of a concentrated charge in a brittle solid medium," Izv. Akad. Nauk ArmSSR, Mekh., 21, No. 5 (1968).
3. A. B. Bagdasaryan, Calculation of the action of an explosion in brittle rock," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1970).
4. V. N. Nikolaevskii, "The connection between volumetric and shear deformations and shock waves in soft soils," Dokl. Akad. Nauk SSSR, 175, No. 5 (1967).
5. É. I. Andrianskii and V. N. Koryavov, "A shock wave in a variably densified medium," Dokl. Akad. Nauk SSSR, 128, No. 2 (1959).
6. A. S. Kompaneets, "Shock waves in a plastic densified medium," Dokl. Akad. Nauk SSSR, 109, No. 1 (1956).
7. E. Fachioli and A.H. Ang, "A discrete Euler model of the propagation of a spherical wave in a compressed medium," in: The Action of a Nuclear Explosion [Russian translation], Mir, Moscow (1971).